

A remark on the muonium to antimuonium conversion in a 331 model

V. Pleitez

*Instituto de Física Teórica
Universidade Estadual Paulista
Rua Pamplona, 145
01405-900– São Paulo, SP
Brazil
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Abstract

Here we analyze the relation between the search for muonium to antimuonium conversion and the 331 model with doubly charged bileptons. We show that the constraint on the mass of the vector bilepton obtained by experimental data can be evaded even in the minimal version of the model since there are other contributions to that conversion. We also discuss the condition for which the experimental data constraint is valid.

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Recently a new upper limit for the spontaneous transition of muonium ($M \equiv \mu^+ e^-$) to antimuonium ($\overline{M} \equiv \mu^- e^+$) has been obtained [1]. This implies constraints upon the models that induce the $M \rightarrow \overline{M}$ transition. One of them is the 331 model proposed some years ago [2]. Here we would like to discuss the conditions in which this constraint can be evaded even in the context of the minimal version of the model (minimal in the sense that no new symmetries or fields are introduced).

In that model in the lepton sector the charged physical mass eigenstates (unprimed fields) are related to the weak eigenstates (primed fields) through unitary transformations ($E_{L,R}$) as follows:

$$l'_L = E_L l_L, \quad l'_R = E_R l_R, \quad (1)$$

where $l = e, \mu, \tau$. It means that the doubly charged vector bilepton, U_μ^{++} , interacts with the charged leptons through the current given by

$$J_{U^{++}}^\mu = -\frac{g_{3l}}{\sqrt{2}} \overline{l'_L} \gamma^\mu \mathcal{K} l_L, \quad (2)$$

where \mathcal{K} is the unitary matrix defined as $\mathcal{K} = E_R^T E_L$ in the basis in which the interactions with the W^+ are diagonal ($\nu'_L = E_L \nu_L$) [3].

In the theoretical calculations of the $M \rightarrow \overline{M}$ transition induced by a doubly charged vector bilepton so far only the case $\mathcal{K} = \mathbf{1}$ has been considered [4]. Although this is a valid simplification it does not represent the most general case in the minimal 331 model. In fact, in that model in the quark sector all left-handed mixing matrices survive in different places of the lagrangian density [5]. In the lepton sector both, left- and right-handed mixing matrices survive in the interactions with the doubly charged vector bilepton as in Eq. (2) and also with doubly and singly charged scalars (see below). Hence, these mixing matrices as \mathcal{K} in Eq. (2) have the same status than the Kobayashi-Maskawa mixing matrix in the context of the standard model in the sense that they must be determined by experiment. In Ref. [1] it is recognized that their bound is valid only for the flavor diagonal bilepton gauge boson case *i.e.*, $\mathcal{K} = \mathbf{1}$. If nondiagonal interactions, like in Eq. (2), are assumed the new upper limit on the conversion probability in the $M \rightarrow \overline{M}$ system implies

$$M_{U^{++}} > g_{3l} |\mathcal{K}_{\mu\mu}| |\mathcal{K}_{ee}| 2.6 \text{ TeV} = 850 |\mathcal{K}_{\mu\mu}| |\mathcal{K}_{ee}| \text{ GeV}. \quad (3)$$

If $|\mathcal{K}_{\mu\mu}| |\mathcal{K}_{ee}| \approx 0.70$ we get a lower bound of 600 GeV for the doubly charged bilepton which is compatible with the upper bound obtained by theoretical arguments [6].

The following is more important. Besides the contribution of the vector bileptons there are also the doubly charged and the neutral scalar ones. To consider only the vector bileptons is also a valid approximation since all the lepton-scalar couplings can be small if all vacuum expectation values (VEVs), except the one controlling the $SU(3)$ breaking, are of the order of the electroweak scale and, if there are no flavor changing neutral currents (FCNC) in the leptonic sector. Both conditions may not be natural in the minimal version of the model. The former because the sextet is introduced only to give mass to the leptons so its VEV may be of the order of a few GeV. The later because if we want to avoid FCNC in the lepton sector it is necessary to impose a discrete symmetry which does not belong to the minimal 331 model. In the model the $SU(3)_L \otimes U(1)_N$ triplets $\eta = (\eta^0 \eta_1^- \eta_2^+)^T \sim (\mathbf{3}, 0)$ and

$\rho = (\rho^+ \rho^0 \rho^{++})^T \sim (\mathbf{3}, +1)$ give mass to the quarks. (The third triplet $\chi \sim (\chi^- \chi^{--} \chi^0)^T$ is out of our concern here.) If the first family transforms in a different way from the other two, the quark u mass is given by the VEV of the η , here denoted by v_η ; if the third family is which transforms differently, it is the quark t which gets its mass from v_η . However, since the mixing matrix of the charge 2/3 quarks is not trivial the general case interpolates between these two cases. Hence, the vacuum expectation values v_η and v_ρ are of the order of the electroweak scale, *i.e.*, $v_\eta^2 + v_\rho^2 = (246 \text{ GeV})^2$. As we said before, the scalar sextet $S \sim (\mathbf{6}, 0)$

$$S = \begin{pmatrix} \sigma_1^0 & \frac{h_2^-}{\sqrt{2}} & \frac{h_1^+}{\sqrt{2}} \\ \frac{h_2^-}{\sqrt{2}} & H_1^{--} & \frac{\sigma_2^0}{\sqrt{2}} \\ \frac{h_1^+}{\sqrt{2}} & \frac{\sigma_2^0}{\sqrt{2}} & H_2^{++} \end{pmatrix} \quad (4)$$

is necessary in order to give to the charged leptons an arbitrary mass. We will denote $\langle \sigma_2^0 \rangle = v_S$ the VEV of σ_2^0 the neutral component of the sextet. The other one, σ_1^0 does not gain a nonzero VEV if the neutrinos must remain massless.

The Yukawa couplings in the lepton sector are

$$- \mathcal{L}_l = \frac{G_{ab}}{\sqrt{2}} \overline{\psi_{aiL}} (\psi_{bjL})^c S_{ij} + \frac{1}{2} \epsilon_{ijk} G'_{ab} \overline{\psi_{aiL}} (\psi_{bjL})^c \eta_k + H.c., \quad (5)$$

where $\psi = (\nu l l^c)^T$ and $a, b = e, \mu, \tau$; i, j are $SU(3)$ indices; G_{ab} and G'_{ab} are symmetric and anti-symmetric complex matrices, respectively. (The model can have CP violation in the leptonic sector [3].)

We stress once more that this is the minimal 331 model because if we want to avoid in Eq. (5) the coupling with the triplet, since only the sextet is necessary for giving all charged leptons a mass, we have to impose a discrete symmetry. (Only in this case there is not FCNC in the lepton sector.) Hence, the mass matrix of the charged leptons has the form

$$M_l = \frac{1}{\sqrt{2}} (G_{ab} v_S + G'_{ab} v_\eta), \quad (6)$$

and it is diagonalized by the bi-unitary transformation $E_L^\dagger M_l E_R = \text{diag}(m_e, m_\mu, m_\tau)$ with E_L and E_R defined in Eq. (1). However, the biunitary transformation does not diagonalize G and G' separately. Thus, we have FCNC and there are Yukawa couplings which are not proportional to the lepton masses.

The model has four singly charged and two doubly charged physical scalars, four CP -even and two CP -odd neutral scalars. Let us consider the doubly charged and neutral scalar Yukawa interactions with the sextet in Eq. (4). H_1^{++} is a part of a complex triplet under $SU(2)_L \times U(1)_Y$ with its neutral partner having vanishing vacuum expectation value (if neutrinos do not get Majorana masses). There are also a doubly charged H_2^{++} which is a singlet of $SU(2)_L$ and the neutral Higgs σ_2^0 which is part of a doublet of $SU(2)_L$. Hence we have the respective Yukawa interactions proportional to

$$\overline{l_L} \mathcal{K}_{LL} l_L^c H_1^{--} + \overline{l_L} \mathcal{K}_{RR} l_R H_2^{++} + [\overline{l_L} \mathcal{K}_{LR} l_R + \overline{l_L}^c \mathcal{K}_{LR}^T l_L^c] \sigma_2^0 + H.c., \quad (7)$$

where we have denoted $\mathcal{K}_{LL} = E_L^T G E_L^*$; $\mathcal{K}_{RR} = E_R^T G E_R$ and $\mathcal{K}_{LR} = E_L^T G E_R$. These matrices are unitary only when G is real.

We see that since the unitary matrices $E_{L,R}$ diagonalize M_l in Eq. (6), \mathcal{K}_{LL} , $\overline{\mathcal{K}}_{RL}$ and \mathcal{K}_{RL} are not diagonal matrices, thus their matrix elements are arbitrary and only constrained by perturbation theory, by their contributions to the charged lepton masses and by some purely leptonic processes.

As we said before, since there are already two scalar triplets which give the appropriate mass to the W^\pm and Z^0 vector bosons, it is not necessary the v_S be of the same order of magnitude than the other vacuum expectation values which are present in the model, v_η and v_ρ . For instance it is possible that $v_S \approx 10$ GeV and $|G| \approx 1$. In this case, the contributions of the doubly charged scalars to the muonium-antimuonium transition can be as important as the contribution of the vector bilepton. There are also new contributions involving FCNC through the neutral scalar exchange, can also give important contributions to the $M \rightarrow \overline{M}$ conversion as it has been suggested in Ref. [7]. We see that in the 331 model all contributions shown in Figs. 1 and 2 do exist. Since there are several contributions to the $M \rightarrow \overline{M}$ conversion it is still possible to have some cancellations among the scalar and vector bilepton contributions.

In order to appreciate a little bit more the muonium-antimuonium transition in the 331 model let us give a brief review of the theoretical results so far known. Many years ago, Feinberg and Weinberg [8] used a $(V - A)^2$ Hamiltonian with the four-fermion effective coupling equal to the usual β -decay coupling constant C_V , in order to study the $M \rightarrow \overline{M}$ conversion. Let us here denote it by $G_{M\overline{M}}$. The transition amplitude is proportional to $\delta = 16G_{M\overline{M}}/\sqrt{2}\pi a^3$ where a is the Bohr radius. More recently, the same transition was studied in the context of models with doubly charged Higgs; in this case the effective Hamiltonian is of the $(V \pm A)^2$ form [9–11]. In this case $G_{M\overline{M}}$ is given by the product of two Yukawa couplings [10,11], so in all these cases the sing of the effective coupling $G_{M\overline{M}}$ is undetermined. On the other hand, in models with doubly charged vector bilepton the respective Hamiltonian is of the $(V - A) \times (V + A)$ form [4] with a four-fermion effective coupling given by $G_{M\overline{M}}/\sqrt{2} = -g^2/8M_U^2$, being M_U the vector bilepton mass and g the $SU(3)$ coupling constant. Hence, in this case always $G_{M\overline{M}} < 0$.

On the other hand, in $(V \pm A)^2$ models the transition amplitude is the same for the singlet and triplet muonium given above [8–10] but in $(V - A) \times (V + A)$ models we have $\delta = -8G_{M\overline{M}}/\sqrt{2}\pi a^3$ for the triplet muonium state and $\delta = 24G_{M\overline{M}}/\sqrt{2}\pi a^3$ for the singlet state [4].

In the 331 model there are also neutral scalars and pseudoscalars which, as we said before, have flavor changing neutral interactions in the lepton sector. It has been shown that pseudoscalars do not induce conversion for triplet muonium, while both pseudoscalars and scalars contribute for the singlet muonium. We see that it is in fact possible a cancellation among the contributions to the $M \rightarrow \overline{M}$ transition due to scalars and those due to doubly charged vector. It means that separate measurements of singlet vs. triplet $M \rightarrow \overline{M}$ conversion probabilities can distinguish among neutral scalar, pseudoscalar and doubly charged Higgs induced transition [7]. Such measurements can also distinguish doubly charged vector bileptons from scalar contributions.

The $M \rightarrow \overline{M}$ transition also can be measured in matter. In this case the collisions make the amplitude add incoherently [8]. However in matter the conversion is strongly suppressed mainly due to the loss of symmetry between M and \overline{M} due to the possibility of μ^- transfer collisions involving \overline{M} [8,12,13]. Hence, all those data together when available will allow to

constrain models with several sort of fields inducing the muonium–antimuonium transition.

If all these effects are present or not in the 331 model depend on the value of the parameters. We have argue above that this may be the fact since there are flavor changing neutral interactions in the Higgs-lepton sector and also one of the vacuum expectation values may be of the order of some GeVs.

It is usually considered that the $\mu \rightarrow e\gamma$ decay imposes stronger constraints on a given model than the $M \rightarrow \overline{M}$ transition. So, some model builders consider situations in which $\mu \rightarrow e\gamma$ is forbidden by a discrete symmetry [11]. However in the 331 model the interactions which induce the $\mu \rightarrow e\gamma$ decay are $\overline{\nu_L} \mathcal{K}_{LR} l_R h_1^+$, $\overline{\nu_L} \mathcal{K}_{LL}^T l_L^c h_2^-$, $[\overline{\nu_L} \mathcal{K}_{LR}' l_R + (\overline{l^c})_L \mathcal{K}_{LR}'^T (\nu_L)^c] \eta_1^-$ and $[\overline{\nu_L} \mathcal{K}_{LL}' (l_L)^c - \overline{l_L} \mathcal{K}_{LL}'^T (\nu_L)^c] \eta_2^+$, with $\mathcal{K}_{LR}' = E_L^\dagger G' E_R$ and $\mathcal{K}_{LL}' = E_L^T G' E_L^*$. The decay $\mu \rightarrow e\gamma$ as shown in Fig. 3 has contributions of the vector U^{--} and scalars $H_{1,2}^{--}$ bileptons. The interactions in Eqs. (2) and (7) involve different mixing matrices, hence, if all bosons have masses of the same order of magnitude, as it is in fact expected in the 331 model (see below), we can have some cancellations among all the contributions. Notice that those matrices are unitary only when G' is real, like the matrices in Eq. (7).

Notice also that the $\mu \rightarrow e\gamma$ decay is dominated by the lepton τ contributions, thus it implies strong constraints on the mixing angles involving this lepton.

Another potential trouble for the model is the $\mu \rightarrow eee$ decay shown in Fig. 4.

Here the amplitudes of the exchange of U^{--} , H_1^{--} and H_2^{--} are proportional to $\mathcal{K}_{\mu e} \mathcal{K}_{ee}$ (\mathcal{K}_{LL}) $_{\mu e} (\mathcal{K}_{RL})_{ee}$ and $(\mathcal{K}_{RL})_{\mu e} (\mathcal{K}_{RL})_{ee}$ respectively, as it can be seen from Eqs. (2) and (7). Thus in this case again a cancellation among the contributions may occur if all bosons have masses of the same order of magnitude, or it may be suppressed by the μe matrix element.

Summarizing, the bound of Ref. [1] is applied only for a range of the parameters in the model and if the sextet is the only Higgs which couples to leptons. In this case the neutral currents given in Eq. (7) are diagonal and there is no FCNC in the lepton sector and all the sextet-lepton couplings are proportional to $\mathcal{K}_{RL} = \sqrt{2} m_l / v_S$ where m_l is the lepton mass. If v_S is of the order of 100 GeV the main contribution to the $M \rightarrow \overline{M}$ conversion comes from the interaction in Eq. (2) and it constrains the mass of the U -vector boson and the mixing angles of the matrix \mathcal{K} as discussed early. Hence in the minimal 331 model the contributions in Fig. 1 (a), 1(c) and 1(d) of Ref. [1], here summarized in Figs. 1 and 2, do exist and its experimental data do not, in straightforward way, apply to the model.

Finally we would like to remark that although the model predicts that there will be a Landau pole at the energy scale μ when $\sin^2 \theta_W(\mu) = 1/4$, it is not clear at all what is the value of μ . In fact, it has been argued that the upper limit on the vector bilepton masses is 3.5 TeV [14]. Any way the important thing is that in this model the “hierarchy problem” *i.e.*, the existence of quite different mass scales, is less severe than in the standard model and its extensions since no arbitrary mass scale (say, the Planck scale) can be introduced in the model. In particular, it is a very well known fact that the masses of fundamental scalars are sensitive to the mass of the heaviest particles which couple directly or indirectly with them. Since in the 331 model the heaviest mass scale is of the order of a few TeVs there is not a “hierarchy problem” at all. This feature remains valid when we introduce supersymmetry in the model. Thus, the breaking of the supersymmetry is also naturally at the TeV scale in this 331 model.

The $M \rightarrow \overline{M}$ transition deserves indeed more experimental studies. On the other hand, the matrices G and G' can be complex, we can have CP violation in the present model [3].

Hence, experimental difficulties apart, this system in vacuum could be useful for studying CP and T invariance in the lepton sector: by comparing $M \rightarrow \overline{M}$ with $\overline{M} \rightarrow M$ transitions as it has been done recently in the $K^0 \rightarrow \bar{K}^0$ and $\bar{K}^0 \rightarrow K^0$ case [15].

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FIGURES

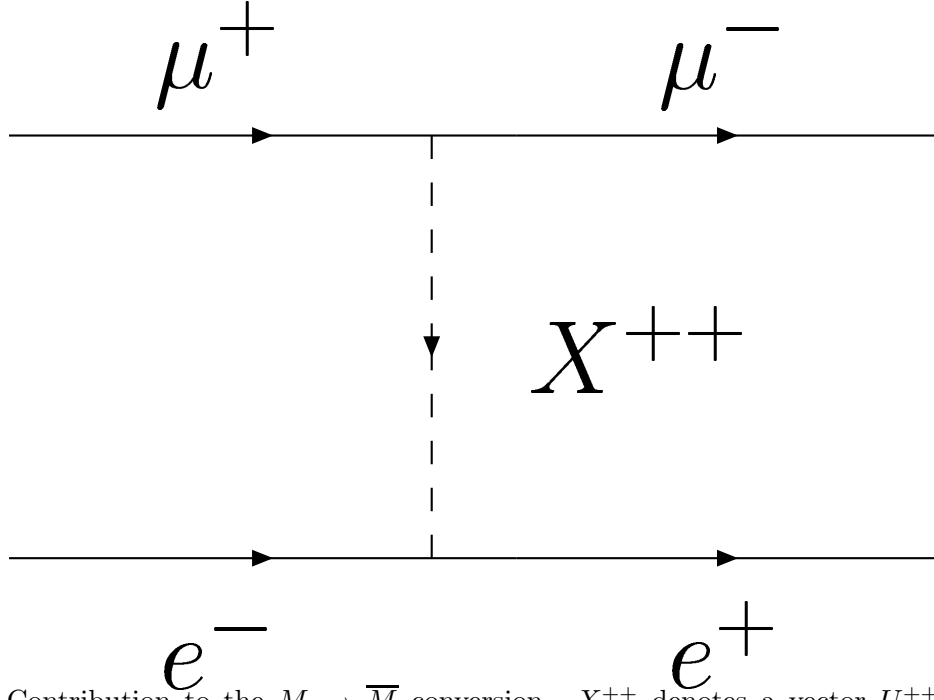


FIG. 1. Contribution to the $M \rightarrow \bar{M}$ conversion. X^{++} denotes a vector U_μ^{++} or a scalar bilepton $H_{1,2}^{++}$.

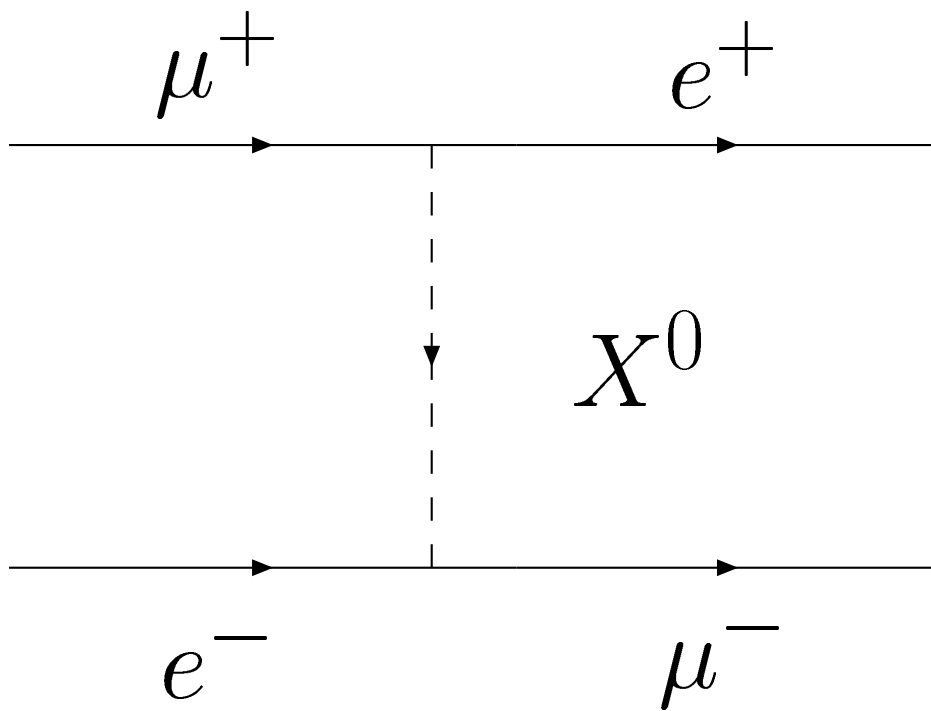


FIG. 2. Contribution to the $M \rightarrow \overline{M}$ conversion. X^0 denotes a neutral scalar or pseudoscalar.

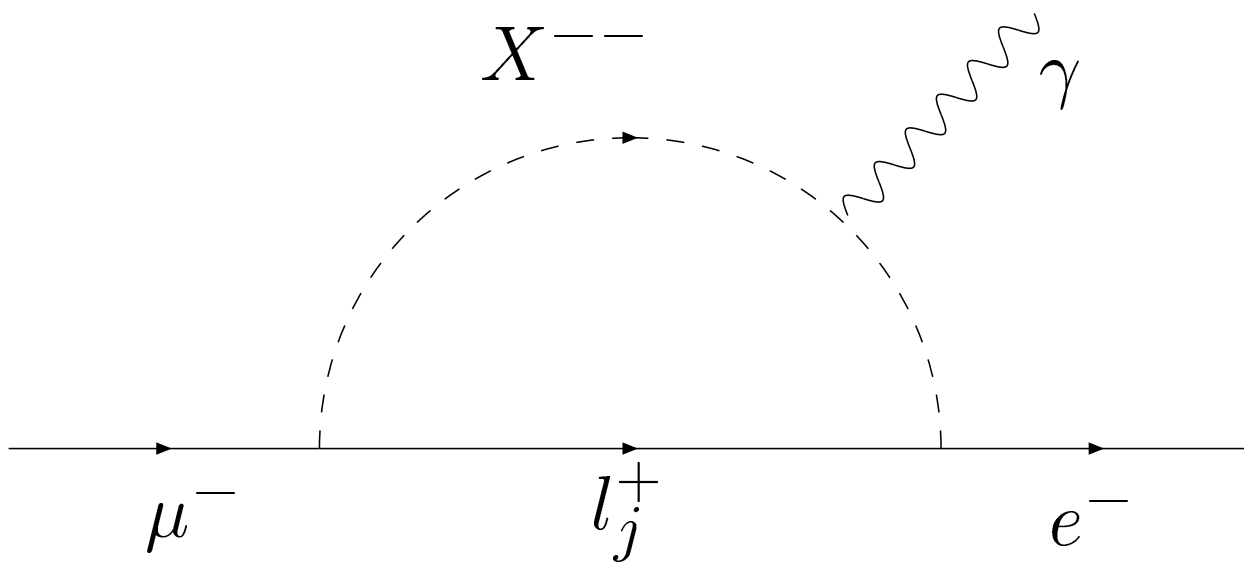


FIG. 3. Contribution to $\mu \rightarrow e\gamma$. As in Fig. 1 X^{--} denotes any of the doubly charged bosons.

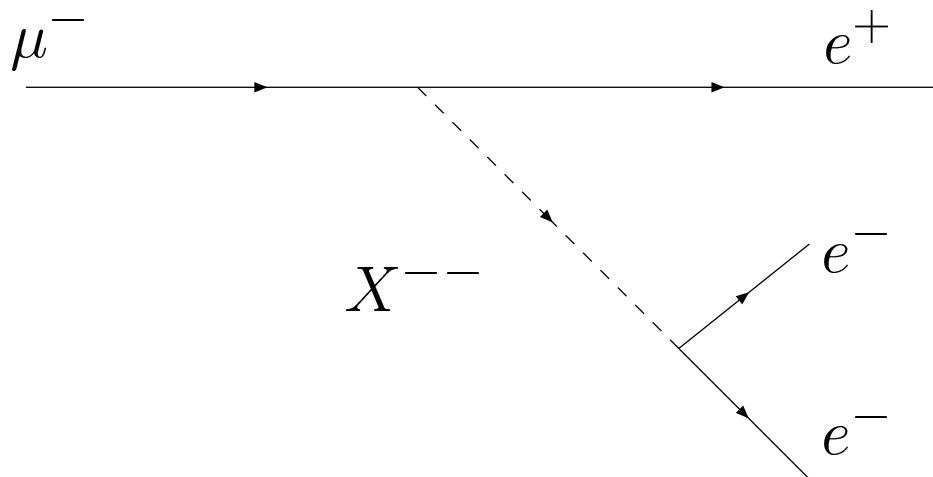


FIG. 4. Contribution to $\mu \rightarrow eee$. As in Fig. 1 X^{--} denotes any of the doubly charged bosons.